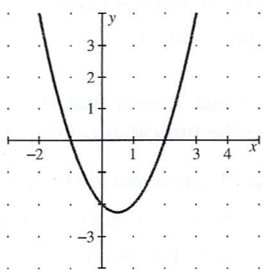
EXERCISES FOR SECTION 3.4

In Exercises 1–10, determine the open intervals on which the graph is concave upward or concave downward.

1. $y = x^2 - x - 2$



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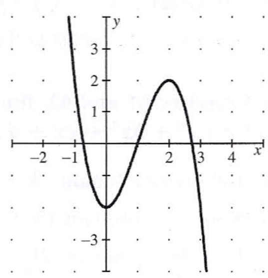
2. $y = -x^3 + 3x^2 - 2$



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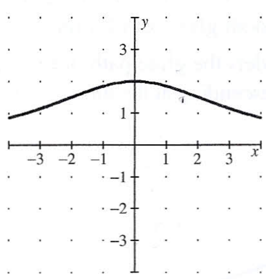
3. $f(x) = \dfrac{24}{x^2 + 12}$



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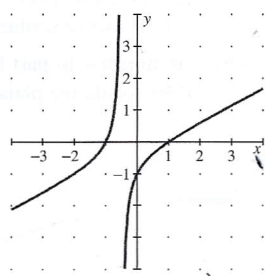
4. $f(x) = \dfrac{x^2 - 1}{2x + 1}$



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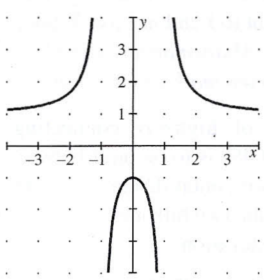
5. $f(x) = \dfrac{x^2 + 1}{x^2 - 1}$



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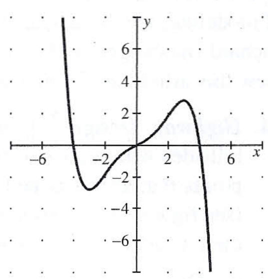
6. $y = \dfrac{-3x^5 + 40x^3 + 135x}{270}$



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7. $g(x) = 3x^2 - x^3$
8. $h(x) = x^5 - 5x + 2$
9. $y = 2x - \tan x, \quad \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$
10. $y = x + \dfrac{2}{\sin x}, \quad (-\pi, \pi)$

In Exercises 11–26, find the points of inflection and discuss the concavity of the graph of the function.

11. $f(x) = x^3 - 6x^2 + 12x$
12. $f(x) = 2x^3 - 3x^2 - 12x + 5$
13. $f(x) = \frac{1}{4}x^4 - 2x^2$
14. $f(x) = 2x^4 - 8x + 3$
15. $f(x) = x(x - 4)^3$
16. $f(x) = x^3(x - 4)$
17. $f(x) = x\sqrt{x + 3}$
18. $f(x) = x\sqrt{x + 1}$
19. $f(x) = \dfrac{x}{x^2 + 1}$
20. $f(x) = \dfrac{x + 1}{\sqrt{x}}$
21. $f(x) = \sin \dfrac{x}{2}, \quad [0, 4\pi]$
22. $f(x) = 2 \csc \dfrac{3x}{2}, \quad (0, 2\pi)$
23. $f(x) = \sec\left(x - \dfrac{\pi}{2}\right), \quad (0, 4\pi)$
24. $f(x) = \sin x + \cos x, \quad [0, 2\pi]$
25. $f(x) = 2 \sin x + \sin 2x, \quad [0, 2\pi]$
26. $f(x) = x + 2 \cos x, \quad [0, 2\pi]$

In Exercises 27–40, find all relative extrema. Use the Second Derivative Test where applicable.

27. $f(x) = x^4 - 4x^3 + 2$
28. $f(x) = x^2 + 3x - 8$
29. $f(x) = (x - 5)^2$
30. $f(x) = -(x - 5)^2$
31. $f(x) = x^3 - 3x^2 + 3$
32. $f(x) = x^3 - 9x^2 + 27x$
33. $g(x) = x^2(6 - x)^3$
34. $g(x) = -\frac{1}{8}(x + 2)^2(x - 4)^2$
35. $f(x) = x^{2/3} - 3$
36. $f(x) = \sqrt{x^2 + 1}$
37. $f(x) = x + \dfrac{4}{x}$
38. $f(x) = \dfrac{x}{x - 1}$
39. $f(x) = \cos x - x, \quad [0, 4\pi]$
40. $f(x) = 2 \sin x + \cos 2x, \quad [0, 2\pi]$

In Exercises 41–44, use a computer algebra system to analyze the function over the indicated interval. (a) Find the first and second derivatives of the function. (b) Find any relative extrema and points of inflection. (c) Graph f, f', and f'' on the same set of coordinate axes and state the relationship between the behavior of f and the signs of f' and f''.

41. $f(x) = 0.2x^2(x - 3)^3, \quad [-1, 4]$
42. $f(x) = x^2\sqrt{6 - x^2}, \quad [-\sqrt{6}, \sqrt{6}]$
43. $f(x) = \sin x - \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x, \quad [0, \pi]$
44. $f(x) = \sqrt{2x} \sin x, \quad [0, 2\pi]$

Getting at the Concept

45. Consider a function f such that f' is increasing. Sketch graphs of f for (a) $f' < 0$ and (b) $f' > 0$.
46. Consider a function f such that f' is decreasing. Sketch graphs of f for (a) $f' < 0$ and (b) $f' > 0$.
47. Sketch the graph of a function f that does *not* have a point of inflection at $(c, f(c))$ even though $f''(c) = 0$.
48. S represents weekly sales of a product. What can be said of S' and S'' for each of the following?
    (a) The rate of change of sales is increasing.
    (b) Sales are increasing at a slower rate.
    (c) The rate of change of sales is constant.
    (d) Sales are steady.
    (e) Sales are declining, but at a slower rate.
    (f) Sales have bottomed out and have started to rise.